

Two exact properties of the perturbative expansion for the two-dimensional electron liquid with Rashba or Dresselhaus spin-orbit coupling

Stefano Chesi^{1,2,3} and Gabriele F. Giuliani¹

¹*Department of Physics, Purdue University, West Lafayette, IN 47907, USA*

²*Department of Physics, University of Basel, 4056 Basel, Switzerland*

³*Department of Physics, McGill University, Montreal, Quebec, Canada H3A 2T8*

(Dated: June 8, 2011)

We have identified two useful exact properties of the perturbative expansion for the case of a two-dimensional electron liquid with Rashba or Dresselhaus spin-orbit interaction and in the absence of magnetic field. The results allow us to draw interesting conclusions regarding the dependence of the exchange and correlation energy and of the quasiparticle properties on the strength of the spin-orbit coupling which are valid to all orders in the electron-electron interaction.

PACS numbers: 71.10.Ca, 71.10.-w, 75.70.Tj, 73.61.Ey

I. INTRODUCTION

Modern yearning for understanding the electronic properties of devices made out of materials in which spins play an important role has rekindled interest in the study of the two-dimensional electron liquid in the presence of Rashba or Dresselhaus spin-orbit coupling.¹⁻³ To date, the effects of the electron-electron interactions in these system have been studied by means of approximated methods, notably the random-phase-approximation (RPA) and the mean-field theory.⁴⁻¹⁴ Quasiparticles parameters were studied in Refs. 4–7 while Refs. 8–14 focused on the ground-state properties and mean-field phase diagram. Recent studies of spatially inhomogeneous Overhauser's type of instabilities (chirality, charge and spin) and of the spin-susceptibility in these systems can be found in Refs. 11 and 13–15, respectively (including non-analytic corrections¹⁵). Calculations of realistic systems of lower dimensionality in the presence of spin-orbit coupling have also been carried out by means of numerical density functional theory methods.^{17,18} However, for want of more accurate exchange and correlation functionals, the latter have been implemented by employing formulas for the energy of the electron liquid obtained in the absence of spin-orbit coupling, an approximation that a priori could appear at best rather crude. Only recently, quantum Monte-Carlo numerical data of the energy were obtained for electrons in two dimensions with Rashba spin-orbit coupling.¹⁶

In this paper we analyze some formal aspects of the perturbative expansion in the electron-electron interaction, without magnetic field and under the assumption that the system will behave as a Fermi liquid. In doing so we will show that certain exact identities lead to the conclusion that, at least in the high density regime or for small spin-orbit coupling strength, the effects of the interactions are only marginally influenced by the spin-orbit coupling. In particular we will show that, to all order in the electron-electron interaction, the corrections to the total energy and the Fermi liquid properties formally vanish to lowest order in the spin-orbit coupling.

As consequence, our results lend some degree of justification and theoretical underpinning for many of the numerical studies carried out by neglecting such effects.

In Ref. 19 a canonical transformation was proposed for the case of small dots that allows one to renormalize the hamiltonian into one in which a generic combination of Rashba and Dresselhaus spin-orbit coupling terms only appears in second order. The vanishing of such spin-orbit interaction effects at small coupling was later verified within the RPA in Ref. 5 for the self-energy. Our analysis not only extends these results to the fully interacting case but also explicitly shows how the repopulation of momentum space with respect to the non interacting situation⁸ is responsible for the renormalization of the Rashba and Dresselhaus coupling. Our formalism also sets the stage for a systematic analysis of the perturbation theory to all order for regimes in which the series is expected to converge. The specific application to the case of Coulomb interaction in the high-density regime and a comparison of analytic formulas for the exchange and correlation energy to the numerical Monte-Carlo results of Ref. 16 can be found in Ref. 12.

The paper is organized as follows: In Section II we provide the definition of the model and the necessary notation; In Section III we prove an exact identity concerning the dependence of the total interacting energy on the suitably defined density dependent bare spin-orbit coupling and the chirality, the relevant Fermi surface repopulation parameter; In Section IV a similar relationship is obtained for the self-energy perturbative series; Finally, Section V provides a discussion of our results.

II. GENERALITIES AND BASIC FORMALISM

Consider the many-body problem associated with the hamiltonian

$$\hat{H} = \sum_i \left[\frac{\hat{\mathbf{p}}_i^2}{2m} + \alpha (\hat{\sigma}_{xi} \hat{p}_{yi} - \hat{\sigma}_{yi} \hat{p}_{xi}) \right] + \frac{1}{2} \sum_{i \neq j} \frac{e^2}{|\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j|}, \quad (1)$$

where the presence of an homogeneous neutralizing background is understood. The second term in the square brackets represents a spin-orbit coupling of the linear Rashba type and can be readily seen to be equivalent to a Dresselhaus spin-orbit coupling term.¹⁻³ While in Eq. (1) the interaction is Coulombic, as it will be shown, our results are valid for a generic interparticle potential. The single particle solutions of the non interacting system are the well known chiral states:

$$\varphi_{\mathbf{k},\pm}(\mathbf{r}) = \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{2L^2}} \begin{pmatrix} \pm 1 \\ ie^{i\phi_{\mathbf{k}}} \end{pmatrix} \equiv \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{L^2}} |\mathbf{k}\pm\rangle \quad (2)$$

where L is the linear size of the system, $\phi_{\mathbf{k}}$ is the angle between the wave vector \mathbf{k} and the x axis and we have defined the spinors $|\mathbf{k}\pm\rangle$.

It is useful at this point to simplify the notation by rescaling all wave vectors by $k_F = \sqrt{2\pi n}$, with n the areal density, and all energies (frequencies) by $\epsilon_F = \frac{\hbar^2 k_F^2}{2m}$ the Fermi energy in the absence of spin-orbit coupling ($\frac{\epsilon_F}{\hbar}$). With the present notation the eigenenergies corresponding to (2) can be simply expressed as

$$\epsilon_{\pm}(k) = k^2 \mp g k, \quad (3)$$

where we have defined the dimensionless coupling constant

$$g = \frac{2m\alpha}{\hbar k_F}, \quad (4)$$

which plays an important role in the following. Without loss of generality, we assume $g \geq 0$.

As discussed in Ref. 9 the unperturbed, spatially uniform states with symmetric k -space occupation can be completely characterized by the generalized chirality χ . This parameter is defined by the Fermi surfaces, two circles with radii given by

$$k_{\pm} = \sqrt{|1 \pm \chi|}. \quad (5)$$

When both chiral bands are occupied this quantity coincides with the regular chirality:

$$\chi = \frac{N_+ - N_-}{N_+ + N_-} < 1, \quad (6)$$

where N_{\pm} is the number of electrons in each chiral band. This is the case for not too low densities and it is the situation we will deal with here.²⁰

The corresponding occupation numbers can then be expressed as follows:

$$n_{\pm}(k) = \theta(\sqrt{|1 \pm \chi|} - k) \quad , \quad \text{for } 0 \leq \chi < 1, \quad (7)$$

where $\theta(x)$ is the usual step function. The non interacting energy per particle (in units of ϵ_F) is in turn given by

$$\mathcal{E}_0(g, \chi) = \frac{1 + \chi^2}{2} - g \frac{\sqrt{|1 + \chi|^3} - \sqrt{|1 - \chi|^3}}{3}. \quad (8)$$

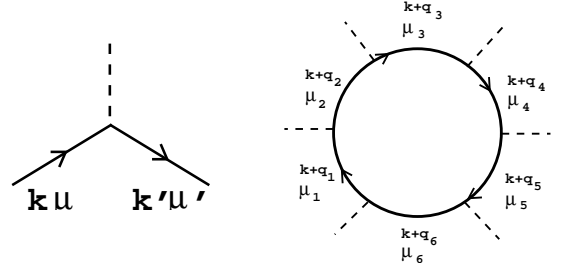


FIG. 1. Left: interaction vertex, associated with a $\langle \mathbf{k}\mu | \mathbf{k}'\mu' \rangle$ factor. Right: example of a fermionic loop.

Here the first term is the kinetic energy while the second one is the spin-orbit coupling contribution.²¹ For non interacting electrons the value of χ that minimizes this expression depends only on the strength of the spin-orbit coupling and the electron density and, remarkably, is in fact uniquely determined by the parameter g [defined above in Eq. (4)] as follows:

$$\chi_0(g) = \begin{cases} g\sqrt{1 - \frac{g^2}{4}} & \text{for } 0 \leq g < \sqrt{2}, \\ \frac{g^2}{4} + \frac{1}{g^2} & \text{for } g \geq \sqrt{2}. \end{cases} \quad (9)$$

The perturbative analysis is very similar to the standard case without spin-orbit coupling.²² The fully interacting Green's function $G_{\mu}(k, t) = -i\langle T \hat{b}_{\mathbf{k}\mu}(t) \hat{b}_{\mathbf{k}\mu}^{\dagger}(0) \rangle$ ($\mu = \pm$) is obtained as an average of the time ordered product of creation (destruction) operators $\hat{b}_{\mathbf{k}\mu}^{\dagger}$ ($\hat{b}_{\mathbf{k}\mu}$) of the chiral states of Eq. (2).²³ The only modification to the diagrammatic rules is that each vertex, beside the delta functions ensuring frequency and momentum conservation, is associated with the scalar product $\langle \mathbf{k}\mu | \mathbf{k}'\mu' \rangle$ [the spinors having been defined in Eq. (2)]. This is exemplified in Fig. 1. Accordingly, the overall dependence of the diagram on the variables χ and g is solely contained in the non interacting Green's functions. The Fourier transform of the latter is given by

$$G_{0\mu}(k, \omega) = \frac{1 - n_{\mu}(k)}{\omega - \epsilon_{\mu}(k) + i\eta} + \frac{n_{\mu}(k)}{\omega - \epsilon_{\mu}(k) - i\eta}, \quad (10)$$

where $n_{\pm}(k)$ is defined in (7) and $\eta = 0^+$ is an infinitesimal quantity. Notice that the dependence on χ is fully contained in the occupation numbers, while the dependence on g is only contained in $\epsilon_{\mu}(k)$. A similar set of diagrammatic rules applies to the self-energy, defined through the relation $G_{\mu}(k, \omega) = 1/(\omega - \epsilon_{\mu}(k) + \Sigma_{\mu}(k, \omega)/\hbar)$.

III. AN EXACT PROPERTY OF THE ENERGY EXPANSION

For any given value of the chirality, the fully interacting total energy per particle can be obtained from the follow-

ing integration over the coupling constant formula:²²

$$\mathcal{E}(g, \chi) = \mathcal{E}_0(g, \chi) + \quad (11)$$

$$+ \pi \sum_{\mu} \int_0^1 \frac{d\lambda}{\lambda} \int \frac{d\mathbf{k}}{(2\pi)^2} \int \frac{d\omega}{2\pi} G_{\mu}^{\lambda}(k, \omega) \Sigma_{\mu}^{\lambda}(k, \omega) ,$$

where the non interacting contribution was given in Eq. (8). In the second term the task is to integrate over λ a sum of closed diagrams, each multiplied by a factor of λ^{n-1} , where n is the order of the interaction. Consider now the expression D of one of these closed diagrams, contributing to the second term of Eq. (11). It is the purpose of this Section to show that

$$\left. \frac{\partial^2 D}{\partial^2 \chi} \right|_0 = \left. \frac{\partial^2 D}{\partial^2 g} \right|_0 = - \left. \frac{\partial^2 D}{\partial \chi \partial g} \right|_0 , \quad (12)$$

where the derivatives are evaluated at $\chi = g = 0$. Our treatment is carried out to all orders of perturbation theory, for a generic diagram D and a general two-body potential $v(q)$.

To prove Eq. (12), we begin by noticing that all the closed diagrams are in general comprised of several fermionic loops. Let us focus our attention on the contribution to Eq. (12) stemming from of one such loop (schematically illustrated in Fig. 1). If the loop contains N solid lines we can write:

$$D = \int [\dots] \sum_{\{\mu_i\}} \int \frac{d\mathbf{k}}{(2\pi)^2} \int \frac{d\omega}{2\pi} \prod_{i=1}^N G_{0\mu_i}(p_i, \omega_i) \times \langle \mathbf{p}_i \mu_i | \mathbf{p}_{i+1} \mu_{i+1} \rangle , \quad (13)$$

where $\{\mu_i\} \equiv \{\mu_1, \mu_2, \dots, \mu_N\}$ and $N+1 \equiv 1$. As in Fig. 1, the internal momenta and frequencies are $\mathbf{p}_i = \mathbf{k} + \mathbf{q}_i$ and $\omega_i = \omega + \Omega_i$ where \mathbf{k} and ω are the momentum and the frequency flowing in the loop. Finally the square bracket $[\dots]$ represents the remaining expression of the diagram, which is independent of \mathbf{k} and ω and does not enter the derivation below.

We proceed by examining the first derivatives of Eq. (13) with respect to the variables χ and g for $g = \chi = 0$. It is readily seen that D depends on χ and g only through the non interacting Green's functions (10). As a consequence, a direct inspection of Eq. (13) leads to the conclusion that both first derivatives for $g = \chi = 0$ are given by a sum of N terms, of which the j -th term contains the first derivative of the corresponding Green's function in the diagram. Since both $\left. \frac{\partial G_{0\mu_j}}{\partial g} \right|_0$ and $\left. \frac{\partial G_{0\mu_j}}{\partial \chi} \right|_0$ are proportional to μ_j , this yields the factor

$$\sum_{\{\mu_i\}} \mu_j \prod_{i=1}^N \langle \mathbf{p}_i \mu_i | \mathbf{p}_{i+1} \mu_{i+1} \rangle = 0, \quad (14)$$

which, as indicated, vanishes upon spin summation.

The second derivatives of D have more complicated expressions that are in general non-vanishing also for $\chi = g = 0$. We observe next that the dependence of

$G_{0\mu}$ on χ and g is such that $\frac{\partial}{\partial \chi}$ only acts on the occupation numbers $n_{\pm}(k)$, while $\frac{\partial}{\partial g}$ only acts on the energy denominators of this function. To prove Eq. (12), we then introduce as a formal device the quantity

$$D_{ab} = 2 \int [\dots] \int \int \frac{d\omega}{2\pi} \frac{d\mathbf{k}_{\chi} d\mathbf{k}_g}{(2\pi)^2} \delta(\mathbf{k}_{\chi} - \mathbf{k}_g) \times \left[\frac{\vec{\nabla}_{\mathbf{k}_a} \cdot \vec{\nabla}_{\mathbf{k}_b}}{4} \right] \prod_{i=1}^N \tilde{G}_0(p_{\chi,i}, p_{g,i}, \omega_i) , \quad (15)$$

where each of the the indexes a, b can be χ or g while $\mathbf{p}_{\chi,i} = \mathbf{k}_{\chi} + \mathbf{q}_i$ and $\mathbf{p}_{g,i} = \mathbf{k}_g + \mathbf{q}_i$. This expression corresponds to the same diagrammatic contribution of Eq. (13). The square parenthesis in the first line coincides with the expression omitted in Eq. (13), but is here evaluated at $g = \chi = 0$. We have also defined the 'resolved' non-interacting Green's function for vanishing spin-orbit coupling ($g = \chi = 0$), i.e.

$$\tilde{G}_0(k_{\chi}, k_g, \omega) = \frac{1 - n_0(k_{\chi})}{\omega - k_g^2 + i\eta} + \frac{n_0(k_{\chi})}{\omega - k_g^2 - i\eta} , \quad (16)$$

where $n_0(k) = \theta(1 - k)$. In this expression, the momenta k_{χ} and k_g respectively appearing in the occupation numbers and the energy denominators are treated as two independent variables. Notice that, except for the differential operator inside the square parenthesis, Eq. (15) coincides with the $\chi = g = 0$ expression of the diagram D . At this point, by integrating by parts (so that the derivatives act on the two dimensional delta function), one can see that

$$D_{\chi\chi} = D_{gg} = -D_{\chi g} . \quad (17)$$

The desired identity Eq. (12) follows from the relation

$$\left. \frac{\partial^2 D}{\partial a \partial b} \right|_0 = D_{ab} , \quad (18)$$

which is proved next.

To derive Eq. (18) we consider first the equivalence of $\left. \frac{\partial^2 D}{\partial \chi^2} \right|_0$ with $D_{\chi\chi}$. The former is the sum of several terms, of which some contain the second derivative of a single Green's function, and others contain the product of first derivatives of two distinct Green's functions. The same holds true for $D_{\chi\chi}$ [see Eq. (15)]. The equality between corresponding second derivative terms of $\left. \frac{\partial^2 D}{\partial \chi^2} \right|_0$ and $D_{\chi\chi}$ can be established by using the relation

$$\left. \frac{\vec{\nabla}_{\mathbf{k}_{\chi}}^2}{4} \tilde{G}_0(p_{\chi}, p_g, \omega) \right|_{p_{\chi}=p_g=p} = \left. \frac{\partial^2 G_{0\mu}(p, \omega)}{\partial \chi^2} \right|_0 , \quad (19)$$

where on the left side we set $p_{\chi} = p_g = p$ and on the right side $g = \chi = 0$. The necessary factor of 2 is obtained performing the spin summation

$$\sum_{\{\mu_i\}} \prod_{i=1}^N \langle \mathbf{p}_i \mu_i | \mathbf{p}_{i+1} \mu_{i+1} \rangle = 2 . \quad (20)$$

For the first derivative terms, we can use the formula ($a = g, \chi$)

$$\left. \frac{\vec{\nabla}_{\mathbf{k}_a}}{2} \tilde{\mathcal{G}}_0(p_\chi, p_g, \omega) \right|_{p_\chi=p_g=p} = - \left. \frac{\partial G_{0\mu}(p, \omega)}{\partial a} \right|_0 \frac{\mu \mathbf{p}}{p}. \quad (21)$$

The factors $\frac{\mathbf{p}_j \cdot \mathbf{p}_k}{p_j p_k}$ are recovered by performing the following spin summations

$$\sum_{\{\mu_i\}} \mu_j \mu_k \prod_{i=1}^N \langle \mathbf{p}_i \mu_i | \mathbf{p}_{i+1} \mu_{i+1} \rangle = 2 \frac{\mathbf{p}_j \cdot \mathbf{p}_k}{p_j p_k}. \quad (22)$$

This establishes the desired result $\left. \frac{\partial^2 D}{\partial \chi^2} \right|_0 = D_{\chi\chi}$.

The equality $\left. \frac{\partial^2 D}{\partial \chi \partial g} \right|_0 = D_{\chi g}$ can be proved in a similar way. For this case, the following relation proves useful

$$\left. \frac{\vec{\nabla}_{\mathbf{k}_\chi} \cdot \vec{\nabla}_{\mathbf{k}_g}}{4} \tilde{\mathcal{G}}_0(p_\chi, p_g, \omega) \right|_{p_\chi=p_g=p} = \left. \frac{\partial^2 G_{0\mu}(p, \omega)}{\partial \chi \partial g} \right|_0. \quad (23)$$

Finally, to prove the equivalence of $\left. \frac{\partial^2 D}{\partial g^2} \right|_0$ and D_{gg} , we use the following formula

$$\left. \frac{\vec{\nabla}_{\mathbf{k}_g}^2}{4} \tilde{\mathcal{G}}_0(p_\chi, p_g, \omega) \right|_{p_\chi=p_g=p} = \left. \frac{\partial^2 G_{0\mu}(p, \omega)}{\partial g^2} \right|_0 + [\mathcal{G}_0(p, \omega)]^2, \quad (24)$$

where $\mathcal{G}_0(p, \omega)$ is the usual non interacting Green's function without spin-orbit coupling ($g = \chi = 0$).²² A slight complication arises, due to the $[\mathcal{G}_0(p, \omega)]^2$ term of (24). This produces an additional contribution of the following form

$$\begin{aligned} & 2 \int [\dots] \int \frac{d\omega}{2\pi} \int \frac{d\mathbf{k}}{(2\pi)^2} \sum_{j=1}^N [\mathcal{G}_0(p_j, \omega_j)]^2 \prod_{i \neq j} \mathcal{G}_0(p_i, \omega_i) \\ & = -2 \int [\dots] \int \frac{d\mathbf{k}}{(2\pi)^2} \int \frac{d\omega}{2\pi} \frac{\partial}{\partial \omega} \prod_{i=1}^N \mathcal{G}_0(p_i, \omega + \Omega_i). \end{aligned} \quad (25)$$

However, (25) is seen to vanish upon integration over the loop frequency ω .

IV. AN EXACT PROPERTY OF THE SELF-ENERGY EXPANSION

The fully interacting self-energy $\Sigma_\mu(k, \omega)$ in the presence of spin-orbit coupling satisfies a similar exact relationship to linear order in g . Again, we consider its diagrammatic expansion and show that for any given diagram D_{Σ_μ} :

$$\left. \frac{\partial D_{\Sigma_\mu}(k, \omega)}{\partial g} \right|_0 = -\frac{\mu}{2} \frac{\partial D_{\Sigma_0}(k, \omega)}{\partial k}. \quad (26)$$

where $\Sigma_0(k, \omega)$ is the interacting self energy without spin-orbit coupling (therefore, D_{Σ_0} is obtained setting $g =$

$\chi = 0$ in D_{Σ_μ}). We will derive Eq. (26) assuming that $\chi \simeq g$ and, as in Eq. (12), the left side derivative is evaluated at $g = 0$. Within the notation of the previous Section, D_{Σ_μ} is written as

$$D_{\Sigma_\mu} = \int [\dots] \sum_{\{\mu_i\}} \prod_{i=1}^N G_{0\mu_i}(p_i, \omega_i) \prod_{j=0}^N \langle \mathbf{p}_j \mu_j | \mathbf{p}_{j+1} \mu_{j+1} \rangle. \quad (27)$$

Here, only Green's functions connected which the external momentum \mathbf{k} are explicitly written. If $1 \leq i \leq N$ we set $\mathbf{p}_i = \mathbf{k} + \mathbf{q}_i$ while $\mathbf{p}_0 = \mathbf{p}_{N+1} = \mathbf{k}$ and $\mu_0 = \mu_{N+1} = \mu$.

Also in this case the derivative $\frac{\partial}{\partial g}$ involves all the non interacting Green's functions appearing in the diagram. On the other hand, as we have discussed in the previous Section, one needs not to worry about the (vanishing) contribution of all the sets of Green's functions that are involved in fermionic loops.

Therefore, the expression of $\left. \frac{\partial D_{\Sigma_\mu}}{\partial g} \right|_0$ consists of the sum of N terms, involving the first derivatives of the Green's functions explicitly appearing in Eq. (27). The final result is

$$\begin{aligned} \left. \frac{\partial D_{\Sigma_\mu}}{\partial g} \right|_0 &= \int [\dots] \sum_{j=1}^N \left(\mu_j \left. \frac{\partial G_{0\mu_j}(p_j, \omega_j)}{\partial g} \right|_0 \right) \mu \frac{\mathbf{k} \cdot \mathbf{p}_j}{k p_j} \\ &\quad \times \mathcal{G}_0(p_1, \omega_1) \mathcal{G}_0(p_2, \omega_2) \dots \mathcal{G}_0(\widehat{p_j}, \omega_j) \dots \mathcal{G}_0(p_N, \omega_N), \end{aligned} \quad (28)$$

where the factor $\mathcal{G}_0(p_j, \omega_j)$ is omitted in the second line. We also notice that, in the first line, the expression in the round brackets is independent of μ_j (since $\left. \frac{\partial G_{0\mu}}{\partial g} \right|_0 \propto \mu$). Furthermore the sums over the spin indexes $\{\mu_i\} \equiv \{\mu_1, \mu_2, \dots, \mu_N\}$ were performed using the relation

$$\sum_{\{\mu_i\}} \mu_j \prod_{i=0}^N \langle \mathbf{p}_i \mu_i | \mathbf{p}_{i+1} \mu_{i+1} \rangle = \mu \frac{\mathbf{k} \cdot \mathbf{p}_j}{k p_j}. \quad (29)$$

We then consider the expression D_{Σ_0} of the same diagram occurring in the expansion of the interacting self energy in the absence of spin-orbit coupling:

$$D_{\Sigma_0} = \int [\dots] \prod_{i=1}^N \mathcal{G}_0(p_i, \omega_i). \quad (30)$$

Its derivative can be directly compared to Eq. (28) by making use of the relation

$$\frac{\partial \mathcal{G}_0(p_j, \omega_j)}{\partial k} = -2\mu_j \left. \frac{\partial G_{0\mu_j}(p_j, \omega_j)}{\partial g} \right|_0 \frac{\mathbf{k} \cdot \mathbf{p}_j}{k p_j}, \quad (31)$$

which is easily verified for $\chi \simeq g$. Eq. (26) then immediately follows.

V. DISCUSSION

We have derived by diagrammatic means two exact relationships involving the total energy expansion and

the self-energy of a two dimensional system in the presence of Rashba or Dresselhaus spin-orbit coupling. The first identity is given in Eq. (12) and involves the second derivatives of the energy diagrams with respect to the density dependent dimensionless spin-orbit coupling g , defined in Eq. (4), and the chirality χ , defined in Eq. (6). Notice that, as made clear in Eq. (5), χ takes into account the renormalized momentum space repopulation brought about by interactions.⁸ Since Eq. (12) is valid for any diagrammatic contribution, we can formally sum the perturbative expansion in the electron interaction. This leads to the following formula, for the small g and χ expression of the extra contribution to the electronic exchange and the correlation energy due to the spin-orbit coupling:

$$\delta\mathcal{E}_{xc}(g, \chi) = C(g - \chi)^2 + \dots, \quad (32)$$

where the constant C is unspecified, and terms of higher order in g and χ are omitted. It is clear from this result that the total energy is an extremum for $\chi \simeq g$. Comparing with Eq. (9) we conclude that to linear order in g the interactions lead to no change in χ , i.e. no repopulation, and there is no energy correction $\delta\mathcal{E}_{xc}(g, \chi)$ to quadratic order in g .¹⁰ This remarkable result does not hold for other types of spin-orbit coupling.⁸

Our second result, Eq. (26), involves the self-energy and allows us to write that to linear order in g

$$\Sigma_\mu(k, \omega) = \Sigma_0(k - \frac{\mu g}{2}, \omega) + \dots \quad (33)$$

From this relation we conclude that, to linear order in g , all the quasiparticles properties (effective mass, lifetime, ...) on the Fermi surfaces $k_\pm \simeq 1 \pm \frac{g}{2}$ can be simply obtained from the case without spin-orbit coupling. Equation (33) extends the results of Ref. 5 where this statement was derived in the RPA approximation for Coulomb interactions. We observe that, similarly to the case of the phonon self-energy in a Fermi liquid, the interaction self-energy for the case of spin-orbit coupling is seen to “ride” the Fermi surface.

Finally, we would like to comment about the relation of our calculation to non-analyticities, which are known for a long time to occur in the theory of the electron liquid^{24,25} and have attracted much attention in more recent years (see, for example, Refs. 15, 26–31). Non-analytic terms in the interactions are fully included in our results, since our derivation applies to all orders in perturbation theory. However, an analytic expansion in the spin-orbit coupling parameters, g and χ , is assumed.

This procedure is justified here because we restrict ourselves to low orders in the spin-orbit coupling parameters (first-order for the self-energy and second order for the total energy). For the total energy, non-analytic corrections can only appear starting from third order in the relevant energy scale, as it can be concluded from a power-counting argument.^{30,31} For example, a contribution to the total energy $\propto |\alpha|E_Z^2$ (where E_Z is the Zeeman energy) is responsible of a non-analytic correction $\propto |\alpha|$ to the spin-susceptibility.¹⁵ This contribution to the energy is vanishing in the absence of magnetic field but a term $\propto |\alpha|^3 \ln^2 |\alpha|$ can be derived at fourth order in the electron interaction.³² A non-analytic correction to the exchange energy is also obtained in Ref. 12, proportional to $\chi^4 \ln |\chi|$. For the self-energy, no terms linear in α were found in Refs. 5 and 14 and non-analytic corrections proportional to $\alpha^2 \ln |\alpha|$ were derived in Ref. 5. All these non-analyticities do not affect our main results, Eqs. (32) and (33), since they only appear at higher order in the spin-orbit coupling.

ACKNOWLEDGMENTS

We would like to thank Dmitrii L. Maslov and Robert A. Žak for helpful discussions and Yuli Lyanda-Geller for pointing us to Ref. 19. SC acknowledges support by NCCR Nanoscience, Swiss NSF, and CIFAR.

¹ Y. A. Bychkov and E. I. Rashba, JETP Lett. **39**, 78 (1984).

² Y. A. Bychkov and E. I. Rashba, J. Phys. C **17**, 6039 (1984).

³ G. Dresselhaus, Phys. Rev. **100**, 580 (1955).

⁴ G.-H. Chen and M. E. Raikh, Phys. Rev. B **60**, 4826 (1999).

⁵ D. S. Saraga and D. Loss, Phys. Rev. B **72**, 195319 (2005).

⁶ I. A. Nechaev, M. F. Jensen, E. D. L. Rienks, V. M. Silkin, P. M. Echenique, E. V. Chulkov, and P. Hofmann, Phys. Rev. B **80**, 113402 (2009).

⁷ I. A. Nechaev, P. M. Echenique, and E. V. Chulkov, Phys. Rev. B **81**, 195112 (2010).

⁸ S. Chesi and G. F. Giuliani, Phys. Rev. B **75**, 155305 (2007).

⁹ S. Chesi, G. Simion, and G. F. Giuliani, cond-

mat/0702060.

¹⁰ S. H. Abedinpour, G. Vignale, and I. V. Tokatly, Phys. Rev. B **81**, 125123 (2010).

¹¹ G. E. Simion and G. F. Giuliani, arXiv:1008.1816.

¹² S. Chesi and G. F. Giuliani, Phys. Rev. B **83**, 235309 (2011).

¹³ S. Chesi, Ph.D. thesis, Purdue University (2007).

¹⁴ A. Agarwal, S. Chesi, T. Jungwirth, J. Sinova, G. Vignale, and M. Polini, Phys. Rev. B **83**, 115135 (2011).

¹⁵ R. A. Žak, D. L. Maslov, and D. Loss, Phys. Rev. B **82**, 115415 (2010).

¹⁶ A. Ambrosetti, F. Pederiva, E. Lipparini, and S. Gandolfi, Phys. Rev. B **80**, 125306 (2009).

¹⁷ M. Valín-Rodríguez, A. Puente, and L. Serra, Phys. Rev. B **66**, 045317 (2002).

- ¹⁸ M. Valín-Rodríguez, A. Puente, L. Serra, and E. Lipparini, Phys. Rev. B **66**, 165302 (2002).
- ¹⁹ I. L. Aleiner and V. I. Fal'ko, Phys. Rev. Lett. **87**, 256801 (2001).
- ²⁰ On the other hand, a value $\chi \geq 1$ corresponds to a low density case in which only the lowest chirality band is occupied, the occupied states forming a ring in momentum space. In this case the regular chirality is simply 1, and differs from the value of χ .
- ²¹ The expression for the case of $\chi \geq 1$ is simply obtained by substituting 2χ for $1 + \chi^2$ in the numerator of the kinetic energy term.
- ²² G. F. Giuliani and G. Vignale, *Quantum Theory of the Electron Liquid* (Cambridge University Press, Cambridge, 2005).
- ²³ For the paramagnetic state, the Green's function is diagonal in μ . We assume \mathbf{k} in the x direction and consider the reflection operator \hat{R}_y , corresponding to $y \rightarrow -y$, which gives $\langle \hat{R}_y^{-1} \hat{b}_{\mathbf{k}\nu} b_{\mathbf{k}\mu}^\dagger \hat{R}_y \rangle = \mu\nu \langle \hat{b}_{\mathbf{k}\nu} \hat{b}_{\mathbf{k}\mu}^\dagger \rangle$. If the ground state has a definite parity with respect to \hat{R}_y , the expectation value should be invariant, and the $\mu\nu = -1$ case gives zero.
- ²⁴ M. Gell-Mann and K. A. Brueckner, Phys. Rev. **106**, 364 (1957).
- ²⁵ A. K. Rajagopal and J. C. Kimball, Phys. Rev. B, **15**, 2819 (1977).
- ²⁶ D. Belitz, T. R. Kirkpatrick, and T. Vojta, Phys. Rev. B **55**, 9452 (1997).
- ²⁷ A. V. Chubukov and D. L. Maslov, Phys. Rev. B **68**, 155113 (2003).
- ²⁸ V. M. Galitski and S. Das Sarma, Phys. Rev. B **70**, 035111 (2004).
- ²⁹ A. V. Chubukov, D. L. Maslov, S. Gangadharaiah, and L. I. Glazman, Phys. Rev. Lett. **95**, 026402 (2005).
- ³⁰ D. L. Maslov, A. V. Chubukov, and R. Saha, Phys. Rev. B **74**, 220402 (2006).
- ³¹ D. L. Maslov, A. V. Chubukov, Phys. Rev. B **79**, 075112 (2009).
- ³² R. A. Žak, D. L. Maslov, and D. Loss, unpublished.